Using extended Kalman filter and least squares method for spacecraft attitude estimation

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Abstract. Herein, the purpose is to compare the attitude estimation results between the least squares method and the extended Kalman filter of an artificial satellite using real data of on-board attitude sensors. These estimation methods are applied for nonlinear problems, where the first is an alternative for the estimation criterion of minimum variance, and yields instantaneous attitude determination, by processing the attitude sensors data. However, the extended Kalman filter carries out the processing of such sensors measurements in real time, and its formulation accounts for the dynamic noise of the states, yielding a kinematic attitude determination, using additionally the gyro measurements. It is observed that the averages of the values of the attitude estimated by the least squares method are very close to the results for the extended Kalman filter. In this way it can be concluded that the algorithm of the extended Kalman filter converges to the least squares solution when fed with real data supplied by the attitude sensors.

1 Introduction

The attitude of a spacecraft is defined by its orientation in space related to some reference system. The importance of determining the attitude is related not only to the performance of attitude control, but also the precise usage of information obtained by payload experiments performed by the satellite.

The attitude estimation is the process of calculating the orientation of the spacecraft in relation to a reference system from data supplied by attitude sensors. Chosen the vectors of reference, an attitude sensor measures the orientation of these vectors with respect to the satellite reference system. Once these one or more vectors measurements are known, it is possible to compute the orientation of the satellite processing these vectors, using methods of attitude estimation.

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There are several methods for determining the attitude of a satellite. Each method is appropriate to a particular type of application and meets the needs as: available time for processing and accuracy to be attained. However, all methods need observations that are obtained by means of sensors installed on the satellite. The sensors are essential for attitude estimation, because they measure its orientation relative to some referential, e.g. the Earth, the sun or a star.

In this work, the satellite attitude is described by Euler angles, due to its easy geometric interpretation, and two methods to estimate the attitude are used: the Extended Kalman Filter and the Least Squares Method. These methods are capable of performing state estimation in nonlinear systems, besides taking into account measurements provided by different attitude sensors. In this work it was considered real data supplied by gyroscopes, infrared Earth sensors and digital sun sensors. These sensors are on board the CBERS-2 satellite (China-Brazil Earth Resources Satellite), and the measurements were collected by the Satellite Control Centre of INPE (Brazilian Institute for Space Research).

2 Representation and estimation of attitude

The attitude of an artificial satellite is directly related to its orientation in space. Through the attitude one can know the spatial orientation of the satellite, since in most cases it can be considered as a rigid body, where the attitude is expressed by the relationship between two coordinate systems, one fixed on the satellite and another associated with a reference system, e.g. inertial system.

In order to that mission present a good performance it is essential that the satellite be stabilized in relation to a specified attitude. The attitude stabilization is done by the on board attitude control, which is designed to acquire and maintain the satellite in a pre-defined attitude. The CBERS-2 attitude is stabilized in three axes and can be described with respect to the orbital system. In this reference system, the movement around the direction of the orbital velocity is called roll (ϕ). The movement around the direction normal to the orbit is called pitch (θ), and finally the movement around the direction Nadir/Zenith is called yaw (ψ).

To transform a vector represented in a given reference to another it is necessary to define a matrix of direction cosines (R), where its elements are written in terms of Euler angles (ϕ, θ, ψ). The rotation sequence used in this work for the Euler angles was the 3-2-1, where the coordinate system fixed in the body of the satellite (x, y, z) is related to the orbital coordinate system (x_o, y_o, z_o) through the following sequence of rotations (Fuming and Kuga, 1999):

- 1st rotation of an angle (yaw angle) around the z_o axis;
- 2nd rotation of an angle (roll angle) around an intermediate axis x’;
- 3rd rotation of an angle (pitch angle) around the y axis.

The matrix obtained through the 3-2-1 rotation sequence is given by:

\[
R = \begin{bmatrix}
C(\theta)C(\psi) & C(\theta)S(\psi) & -S(\theta) \\
S(\phi)S(\theta)C(\psi) - S(\psi)C(\phi) & S(\phi)S(\theta)S(\psi) + C(\phi)C(\psi) & S(\phi)S(\theta)C(\psi) \\
C(\phi)S(\theta)C(\psi) + S(\phi)S(\theta)S(\psi) & -S(\phi)S(\theta)C(\psi) & C(\phi)S(\theta)C(\psi) + S(\phi)S(\theta)S(\psi)
\end{bmatrix} \quad (2.1)
\]

where \( R \) is the matrix of direction cosines with: S=sin, C=cos, T=tan.

By representing the attitude of a satellite with Euler angles, the set of kinematical equations are given by (Wertz, 1978; Fuming and Kuga, 1999):

\[
\begin{bmatrix}
\dot{\phi} \\
\dot{\theta} \\
\dot{\psi}
\end{bmatrix} = \begin{bmatrix}
1 & S(\phi)T(\theta) & C(\phi)T(\theta) \\
0 & C(\phi) & -S(\phi) \\
0 & S(\phi)/C(\theta) & C(\phi)/C(\theta)
\end{bmatrix} \begin{bmatrix}
w_x \\
w_y \\
w_z
\end{bmatrix} \quad (2.2)
\]
where $\omega_0$ is the orbital angular velocity and $\hat{\omega}_x$, $\hat{\omega}_y$, $\hat{\omega}_z$ are the components of the angular velocity on the satellite system.

3 The measurements system of satellite

In order to measure the satellite attitude accurately, several types of sensor, including gyros, earth sensors and solar sensors, are used in the measurement system. The measurement equations of these sensors are introduced here.

3.1 The model for gyros

The advantage of a gyro is that it can provide the angular displacement and/or angular velocity of the satellite directly. However, gyros have an error due to drifting, meaning that their measurement error increases with time. In this work, the rate-integration gyro (RIG) is used to measure the angular velocities of the roll, pitch and yaw axes of the satellite. The mathematical model of the RIGs is (Wertz, 1978):

$$\Delta \Theta_i = \int_0^{\Delta t} (\omega_i + \epsilon_i) \, dt \quad (i = x, y, z)$$

(3.1)

where $\Delta \Theta$ are the angular displacement of the satellite in a time interval $\Delta t$, and $\epsilon_i$ are components of bias of the gyroscope.

Thus, the estimated components of the angular velocity of the satellite are given by:

$$\hat{\omega} = \left(\frac{\Delta \Theta}{\Delta t}\right) - \hat{\epsilon} - \eta_1 = \mathbf{g} - \epsilon - \eta_1$$

(3.2)

where $\mathbf{g}(t)$ is the output vector of the gyroscope; and $\eta_1(t)$ represents a Gaussian white noise process.

3.2 The measurement model for infrared earth sensors (IRES)

One way to compensate for the drifting errors present in gyros is to use the earth sensors. These sensors are located on the satellite and aligned with their axes of roll and pitch. In the work, two earth sensors are used, with one measuring the roll angle and the other measuring the pitch angle. In principle, an earth sensor can not measure the yaw angle.

The measurement equations for the earth sensors are given as (Fuming and Kuga, 1999):

$$\Phi_H = \phi + b_\phi$$
$$\Theta_H = \theta + b_\theta$$

(3.3)

where $b_\phi$ and $b_\theta$ are the biases representing the misalignment, installation and/or assembly errors.

3.3 The measurement equation for digital solar sensors (DSS)

Since an earth sensor is not able to measure the yaw angle, the solar sensors are used by the Attitude Control System in order to get around this problem. However, these sensors do not provide direct measurements but coupled angle of pitch ($\alpha_0$) and yaw ($\alpha_w$). The measurement equations for the solar sensor are obtained as follows (Fuming and Kuga, 1999):
\[
\alpha_y = \tan^{-1} \left( \frac{-S_y}{S_x \cos 60^\circ + S_z \cos 150^\circ} \right), \text{when} \quad |S_x \cos 60^\circ + S_z \cos 150^\circ| \geq \cos 60^\circ
\]

\[
\alpha_\theta = 24^\circ - \tan^{-1} \left( \frac{S_x}{S_z} \right), \text{when} \quad 24^\circ - \tan^{-1} \left( \frac{S_x}{S_z} \right) < 60^\circ
\] (3.4)

The conditions are such that the solar vector is in the field of view of the sensor, and \(S_x, S_y, S_z\) are the components of the unit vector associated to the sun vector in the satellite system and given by:

\[
\begin{align*}
S_x &= S_{ox} + \hat{\psi} S_{oy} - \hat{\theta} S_{oz} \\
S_y &= S_{oy} - \hat{\psi} S_{ox} + \hat{\phi} S_{oz} \\
S_z &= S_{oz} - \hat{\phi} S_{oy} + \hat{\theta} S_{oz}
\end{align*}
\] (3.5)

where \(S_{ox}, S_{oy}, S_{oz}\) are the components of the sun vector in the orbital coordinate system (Fuming and Kuga, 1999) and \(\hat{\phi}, \hat{\theta}, \hat{\psi}\) are the Euler angles estimated attitude.

4 Attitude estimation methods

The goal of an estimator is to calculate the state vector (attitude) based on a set of observations (sensors). In other words, it is an algorithm capable of processing measurements to produce, according with a given criterium, a minimum error estimate of the state of a system. In this work, to estimate the state vector, we used two different methods, which follows.

4.1 Extended Kalman filter

The solution using the Kalman Filter is given in two stages, i.e., prediction phase and update phase. To this end, we assume that the dynamics of state and observations are described by nonlinear differential equations with reference to the state given by the system below:

\[
\begin{align*}
y_k &= h_k(x_k) + \nu_k \\
\dot{x} &= f(x, t) + Gw
\end{align*}
\] (4.1)

These equations must be linearized so that we can make use of the linear Kalman Filter. This linearization is done using a Taylor series expansion around the best estimate of the available state. Thus, in the linearization the function \(f\) is truncated to its first derivative, in the form below:

\[
f = f(\bar{x}, t) + \left[ \frac{\partial f}{\partial x} \right]_{x = \bar{x}} (x = \bar{x})
\] (4.2)

We define the deviations as:

\[
\begin{align*}
\delta x &= x - \bar{x} \\
\delta \dot{x} &= \dot{x} - \bar{x} = \dot{x} - f(\bar{x}, t)
\end{align*}
\] (4.3)

and \(F = \left[ \frac{\partial f}{\partial x} \right]_{x = \bar{x}}\), so that \(\delta \dot{x} = F \delta x + Gw\).

The equation of observations is linearized by expanding \(h_k\), also in Taylor series with truncation in the linear term:

\[
y_k = h_k(\bar{x}) + \left[ \frac{\partial h_k}{\partial x} \right]_{x = \bar{x}} (x - \bar{x}) + \nu_k
\] (4.4)
The deviations are defined by
\[ \delta y_k = y_k - h_k(\bar{x}_k) \] and
\[ H_k = \left[ \frac{\partial h_k}{\partial x} \right]_{x=\bar{x}}, \] so that finally the equation of the observations is given by the linearized equation:
\[ \delta y_k = H_k \delta x_k + v_k \] (4.5)

Thus, the equations of prediction phase of the state and the transition matrix are:
\[ \dot{\bar{x}} = f(\bar{x},t) \] and \[ \dot{\Phi} = F \Phi \] (4.6)

The covariance matrix is predicted through the equation:
\[ \bar{P}_k = \Phi_{k-1,k} \hat{P}_{k-1} \Phi_{k-1,k}^T + \int_{k-1}^{k} \Phi_{k-1,k} G(\tau) Q(\tau) G^T(\tau) \Phi_{k-1,k}^T d\tau \] (4.7)

For the update phase we have the equations below:
\[ K_k = \hat{P}_k H_k^T (H_k \hat{P}_k H_k^T + R_k)^{-1} \] (4.8)
\[ \hat{x}_k = \bar{x}_k + K_k [y_k - h_k(\bar{x}_k)] \] (4.9)
\[ \hat{P}_k = (I - K_k H_k) \hat{P}_k \] (4.10)

### 4.2 Instantaneous Least Squares Method

It is an alternative to the criterium of minimum variance estimation. This approach requires an assumption about the non-statistical sources of uncertainty in the problem, assuming the dynamics of the state accurately, i.e., without noise.

The nonlinear least squares estimator assumes the following measurement system:
\[ y_k = h_k(x_k) + v_k \] (4.11)

Consider the a-priori information at the time \( t_0 \) where we want to estimate the state, given by:
\[ \hat{x}_0 = \hat{x}_0(t_0) \] and \( P_0 = \hat{P}_0(t_0) \).

Because the nonlinear least squares are implemented iteratively, then the deviations are refined rather than the state, where:
\[ \delta \hat{x}_k = \hat{x}_k - \hat{x}_0 \]
\[ \delta \bar{x}_k = \bar{x}_k - \bar{x}_{k-1} \] (4.12)

Therefore, the equations that implement the algorithm are given by:
\[ \hat{P}_k = (\hat{P}_0^{-1} + H^T R^{-1} H)^{-1} \]
\[ \delta \hat{x}_k = \hat{P}_k(\hat{P}_0^{-1} \delta \bar{x}_{k-1} + H^T R^{-1} \delta y) \]
\[ \hat{x}_k = \bar{x}_{k-1} + \delta \hat{x}_k \] (4.13)
5 Results

The results presented below are based on real data and show the behavior of each component of the estimated state vector, the error associated with measurements, as well as the estimated covariance.

The satellite used to test the performance of the estimators was the CBERS-2, launched on October 21, 2003. The measurements are for the month of April 2006, with a sampling rate of about 8.56 sec.

To check the performance the Extended Kalman Filtering was exercised to estimate the satellite attitude, and this was then compared the Least Squares Method results, considering the following set of initial conditions:

- Initial Attitude: $\phi_0 = \theta_0 = \psi_0 = 0^\circ$;
- Initial Bias of Gyro: $\varepsilon_x = 5.56^\circ/h$; $\varepsilon_y = 0.87^\circ/h$; $\varepsilon_z = 6.12^\circ/h$;
- Initial Covariance $(P)$: $(P_{att})^2 = (0.5^\circ)^2$ (error related to the attitude); $(P_{bg})^2 = (1^\circ/h)^2$ (error related to the drift of gyro);
- Observation error Covariance $(R)$: $\sigma_{DSS}^2 = (0.3^\circ)^2$ (sun sensor); $\sigma_{IRES}^2 = (0.03^\circ)^2$ (earth sensor);
- Dynamic Noise Covariance $(Q)$: $\sigma_{att}^2 = (0.1^\circ)^2$ (noise related to the attitude); $\sigma_{bg}^2 = (0.01^\circ/h)^2$ (noise related to the drifting of gyro);

It is observed in Fig. 1 that the behavior of attitude during the period analyzed is as expected, with the average estimated values for the axes of roll and pitch, considering the Extended Kalman Filter, are in the order of -0.49 $^\circ$ and -0.41 $^\circ$, respectively and their standard deviations are about 0.05 $^\circ$. For the yaw axis the estimate seems not to behave randomly and its standard deviation is 0.3 $^\circ$. The Least Squares Method estimated only the attitude, and their values are very close to those obtained by the filter. For the roll and pitch axes, the values are approximately -0.46 $^\circ$ and the yaw axis the value is about -1.5 $^\circ$.

![Fig. 1](image1.png)

**Fig. 1** Behavior of estimated attitude by Extended Kalman Filter and the Least Squares Method.

In Fig. 2, we observe the behavior of the components of the gyro biases estimated only by Extended Kalman Filter. The standard deviation of the components of the bias of the gyro for each axis is in the order of $10^{-2}\circ/h$ for x axis and $10^{-2}\circ/h$ for y and z axes.
In Fig. 3, we can see that the residues of Earth sensors and Sun sensors have the same behavior for both methods. These results are consistent because in this case it is not possible to compare the estimated values with true values, since these values are not known.

Fig. 2  \textit{Biases estimated by Extended Kalman Filter.}

Fig. 3  \textit{Residues related to the measures obtained by DSS (Digital sun sensor) and IRES (earth sensor), with the Extended Kalman Filter and Least Squares Method.}

The Fig. 4 presents the estimated variances for both methods for the attitude, and the bias of the gyro estimated by the filter. It is observed that the attitude variance decreases with a tendency to
stabilize around a value. For the variance of the gyro bias there is a decrease in values indicating that it has not attained steady state in this data set.

![Estimated state error (variances)](image)

**Fig. 4** Estimated state error (variances).

### 6 Conclusions and future work

The main objective of this study was to estimate the attitude of a CBERS-2 like satellite, using real data provided by sensors that are on board the satellite. To verify the consistency of the estimator, the attitude was estimated by two different methods.

The usage of real data from on-board attitude sensors, poses difficulties like mismodelling, mismatch of sizes, misalignments, unforeseen systematic errors and post-launch calibration errors. However, it is observed that the attitude estimated by the least squares method are in close agreement with the results of the Extended Kalman Filter (EKF). It can be concluded that the algorithm of the EKF converges to the least squares solution, providing a kinematic attitude solution besides estimating biases (gyro drifts).

In future work, the attitude can be estimated using the real time estimator named Unscented Kalman Filter (UKF). This new approach to the Kalman filter is applicable to nonlinear problems, and tries to avoid undesired linearizations needed otherwise like in EKF.

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