KALMAN FILTER APPLICATION FOR STATE ESTIMATION OF AEROASSISTED MANEUVER

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Abstract: This work presents a simulation of an aeroassisted maneuver around the Earth using a Kalman filter for estimating the position and velocity of the spacecraft at each step. The simulator developed considers a reference trajectory and a trajectory perturbed by external disturbances combined with non-idealities of sensors and actuators. It is able to operate in closed loop controlling the trajectory at each instant of time using a PID controller and propulsive jets. A Kalman filter utilizes the sensor data to estimate the timewise state of the spacecraft. The estimation algorithms and propagation equations used in this process are presented in this paper. The U.S. Standard Atmosphere is adopted as the atmospheric model. The main results are compared with the case that the Kalman filter is not used. Therefore, it was possible to perform an analysis of the Kalman filter importance during an aeroassisted maneuver.

Key words: Kalman filter, aeroassisted maneuver, trajectory control

1. INTRODUCTION

An orbital maneuver is the transfer of a satellite from one orbit to another by means of a change in velocity. To perform this changing, the spacecraft have to engage the thrusters or use the natural forces of the environment. The Hohmann transfer and Bi-elliptical transfer are some alternatives to perform an orbital maneuver by propulsive means. In 1961, Howard London presented the approach of using aerodynamic forces to change the trajectory and velocity of a spacecraft, this new technique became known as aeroassisted maneuvers (Walberg, 1985). This type of orbital transfer can be accomplished in several layers of the atmosphere. The altitude reached by vehicle is linked to the mission purpose and maximum thermal load supported by the vehicle structure. The main advantage of this type of maneuver is the fuel economy. According to Walberg (1985), many papers on aeroassisted orbital transfer have been made in recent decades and have been shown that a significant reduction in fuel can be achieved using aeroassisted maneuvers instead of Hohmann transfer.

The transfer between two circular and coplanar orbits is very common. The technique of using atmospheric drag to reduce the semi-major axis got known as aerobraking and was first used on March 19, 1991 by spacecraft Hiten. The launch was conducted by the Institute of Space and Astronautical Science of Japan (ISAS). The spacecraft passed through Earth’s atmosphere at an altitude of 125.5 km over the Pacific Ocean at a speed of 11 km / s. The experience resulted in a decrease in apogee altitude of 8665 km. In May 1993, an aerobraking maneuver was used on a mission to Venus by Magellan spacecraft, whose goal was circularize the orbit of the spacecraft. In 1997, the probe U.S. Mars Global Surveyor (MGS) has used its solar panels as “wings” to control its passage through the tenuous upper atmosphere of Mars and lower its apoapsis.

In this work, it was used the Aerobraked Spacecraft Maneuver Simulator (SAMS) with the implementation of the Kalman filter. The SAMS was based in an orbital maneuver simulator developed by Rocco (2006). Usually, it is used an open loop control controlled by land for correction maneuver and orbit transfer. However, in some missions like drag-free (Gravity Probe B and Hipparcos) the feedback control is mandatory. The SAMS considers a reference trajectory and a trajectory perturbed by external disturbances, including the aerodynamic effects, combined with non-idealities of sensors and actuators. It is able to operate in closed loop controlling the trajectory at each instant of time, which is one of the input parameters, using a PID controller and propulsive jets. Gomes (2011) uses the SAMS to present the study of how the orbital elements can be changed by an aeroassisted maneuver and how much of fuel is saved comparing with a propulsive maneuver.

The Kalman filter is a tool that can estimate the variables of a wide range of processes and of all possible filters it is the one that minimizes the variance of the estimation error. It is an estimator with real-time characteristics, i.e., it provides estimates for the moment that the measure is processed (Maybeck, 1979).
Fig. (1) shows a basic diagram about the running logic of the aeroassisted maneuver simulator.

![Diagram of the aeroassisted maneuver simulator](image)

**Figure 1. Basic diagram of the aeroassisted maneuver simulator.**

This article will show a simulation of an aerobraking maneuver around the Earth using a Kalman filter for estimating the position and velocity of the spacecraft at each step. After each passage by atmospheric region, occurs the decrease of the apogee altitude. In this paper we adopted a spacecraft with a cubic body composed of two rectangular plates, called aerodynamic plates, placed in opposite sides of the vehicle body. The inclination angle of the plates with respect to the molecular flow is called of attack angle, whose value has been placed at 90 degrees to maximize the projected area and the drag force.

### 2. METHODS

The spacecraft state is described by the coordinates $X = [X Y Z \dot{X} \dot{Y} \dot{Z}]$ measured in an inertial frame centered on Earth. The spacecraft trajectory is calculated by solving the Kepler equation. The sensor was modeled so that their measurements show a random error with zero mean and variance adjusted by the user of the simulator.

The dynamic model is composed of the gravitational acceleration and the acceleration of the atmospheric drag as shown in the Eq. (1):

$$\ddot{r} = -\frac{\mu}{r^3} r - \frac{1}{2} C_D \rho \frac{S}{M} V_r V_r + w$$

where $\mu$ is the gravitational parameter (product of the central body mass and the universal gravitational constant); $C_D$ is the drag coefficient; $\rho$ is the atmospheric density; $S$ is the projected area; $M$ is the spacecraft mass; $V_r$ is the velocity of the spacecraft relative to the atmosphere and $w$ is the process noise. The dynamic noise $w$ is modeled by a white process whose statistics is given by $w = N(0,Q)$, i.e., zero mean and a covariance $Q$. This noise is applied to the inertial position of the spacecraft.

The atmospheric model U.S. Standard Atmosphere provides the value of the atmospheric density, depending on the position of the vehicle, for the calculation of aerodynamic forces. The velocity of the spacecraft relative to the atmosphere in the inertial system is calculated assuming that the atmosphere has the same rotation velocity of the Earth and its equation is given by Kuga et. al. (2011) and shown in the Eq. (2):

$$V_r = \dot{r} \times r = \begin{bmatrix}
\dot{x} + \omega y \\
\dot{y} - \omega x \\
\dot{z}
\end{bmatrix}$$

where $\dot{r}$ is the velocity vector relative to the inertial system and $\omega$ is the angular velocity of Earth's rotation.

A PID controller was used to correct the deviation in the trajectory. Most of the industrial controllers are Proportional-Integral-Derivative (PID) due to its flexibility, low cost and robustness. The Eq. (3) shows the control law of a PID controller:

$$c(t) = K_p e(t) + K_i \int e(t) dt + K_d \frac{de(t)}{dt}$$

where $K_p$, $K_i$ and $K_d$ are the proportional gain, integral gain and derivative gain respectively and $e(t)$ is the position error. The observation model is given in the Eq. (4):

$$y = h(x) + v$$
where \( y \) is the measurement vector; \( h(x) \) is a non-linear function of the state vector and \( v \) is a vector of discrete white noise whose statistics is given by \( v = N(0, R) \), i.e., zero mean and a covariance \( R \). The measurement noise is present in the sensor reading of the spacecraft inertial position.

The extended Kalman filter was used because the model is non-linear. This filter generates some reference trajectories that are updated at each measurement processing. The filtering process consists of two steps: time-update and measurement-update. Equation (5) shows the time-update process:

\[
\hat{x} = f(\hat{x})
\]
\[
\tilde{P} = FF^T + GQG^T
\]

where \( F \) is the jacobian matrix of \( f \) with respect to \( x \); \( P \) is covariance matrix and \( \hat{x} \) is the propagated state vector. The Eq. (6) shows the measurement-update process:

\[
K = \tilde{PH}^T (H\tilde{P}H^T + R)^{-1}
\]
\[
\hat{P} = (I - KH)\tilde{P}
\]
\[
\hat{x} = \hat{x} + K[y - h(\hat{x})]
\]

where \( K \) is the Kalman gain; \( H \) is the jacobian matrix of \( h(x) \) with respect to \( x \) and it models how the observations are connected with the state and \( \hat{x} \) is the estimated state vector (Maybeck, 1979). The estimated state is used by the PID controller to correct the trajectory error.

3. RESULTS

This section aims to present the results of an aeroassisted maneuver simulation around the Earth using a Kalman filter for estimating the position and velocity of the spacecraft at each step. It was adopted a spacecraft of 500 kg of mass, a cubic body of 1m in each side and two aerodynamic plates of 2m in length. The step used in the simulation was of 1 second and it was performed up to eight hours of maneuvering. At each step, the control system (PID controller) seeks to correct the disturbed trajectory by sending a control signal to the propulsion system that can apply a thrust of up to 20N per second. The specific impulse considered was of 460s that corresponds to the liquid oxygen/liquid hydrogen propellant. Table (1) shows the initial conditions of the orbit.

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Apogee altitude (km)</td>
<td>1000</td>
</tr>
<tr>
<td>Perigee altitude (km)</td>
<td>120</td>
</tr>
<tr>
<td>Eccentricity</td>
<td>0.0634</td>
</tr>
<tr>
<td>Inclination (degrees)</td>
<td>1</td>
</tr>
<tr>
<td>RAAN(^1) (degrees)</td>
<td>200</td>
</tr>
<tr>
<td>Perigee argument (degrees)</td>
<td>10</td>
</tr>
<tr>
<td>Mean anomaly (degrees)</td>
<td>180</td>
</tr>
</tbody>
</table>

Note: \(^1\) Right Ascension of Ascending Node

The spacecraft position is measured with an error of 0.5 meters (one standard deviation) and the dynamic noise has a standard deviation of 0.01 meters. After each passage by atmospheric region, occurs the reducing of the apogee of the transfer orbit. Figure (2) shows the orbit of the maneuver in the XY plane and Fig. (3) presents the spacecraft altitude versus time.
In both figures we can see the reduction of the apogee altitude whose final value was 857.30 km, i.e., a reduction of 142.70 km. The perigee altitude remains at around 120 km. There was no change in the orbit inclination because lift forces were not being applied to the vehicle. The estimated state is composed by position and velocity and is shown in the Fig. (4). On the left side there is the estimated position of the X, Y and Z coordinates and on the right side we have the estimated velocity. Although we have estimated the position and velocity, the controller uses only the estimated position to correct the trajectory.
Figure (5) illustrates the deviation of the position as function of time. This divergence occurs due to the measurement errors, causing a deviation between the reference trajectory and the disturbed trajectory. During the maneuver, the control system acts to reduce this divergence. On the left side of the figure we have the results without the use of Kalman filter and on the other side we have the results with the Kalman filter. The deviation in the first case has reached values of approximately 2 m while the second case the deviation was of about 0.15 m.

![Figure 5. Deviation in the position (with and without Kalman filter) as function of time.](image)

We can also evaluate the deviation of the semi-major axis as function of time shown in the Fig. (6). The figure with blue line shows the results without the Kalman filter and the figure with green line presents the results with the application of the filter. The deviation in the first case has reached values of approximately 300 m while the second case the deviation was of about 35 m.

![Figure 6. Deviation in the semi-major axis (with and without Kalman filter) as function of time.](image)

Figure (7) presents the residue in the X component of the position vector. The figures of the other components of the position vector are similar and therefore were omitted. In the first graph the results are presented for the complete maneuver (the time is shown in seconds) while the second graph the residue are showed up to 3 minutes of maneuver in order to facilitate the visualization of the transient state (the time is shown in minutes). The blue line represents the error between the propagated position and the measured position; the green line represents the error between the propagated position and the Kalman filter’s estimated position and the red line is 2 standard deviations. The estimation position error stays within about 0.5 meters while the measurement error occasionally spikes up to 2 meters.

![Figure 7. Deviation in the position vector residue (X component) as function of time.](image)
Figure 7. Residue in the X component of the position vector.

The final analysis of this study refers to the consumption of propellant between the maneuvers with and without the Kalman filter; these values can be seen in the Tab. (2).

Table 2. Consumption of propellant between the maneuvers performed.

<table>
<thead>
<tr>
<th>Maneuver</th>
<th>Consumption of propellant (kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Without Kalman filter</td>
<td>71.54</td>
</tr>
<tr>
<td>With Kalman filter</td>
<td>14.61</td>
</tr>
</tbody>
</table>

The filter provided a fuel savings of about 57 kg. The propellant was used to correct the trajectory between the reference state and disturbed state. If we improve the measurements accuracy, then we have lower fuel consumption as presented by Oliveira et. al (2011).

4. CONCLUSIONS

It was presented the simulation of an aeroassisted maneuver around the Earth with the implementation of the Kalman filter to estimate the position and velocity of the spacecraft. A PID controller and propulsive jets were used to correct the deviation between the reference trajectory and the disturbed trajectory. This study aimed to show the importance of the Kalman filter in an aeroassisted maneuver and this goal was achieved through the simulations presented in the last section. The results show that the Kalman filter reduces the position error and so we have a significantly economy fuel. Further tests indicated that when the measurement error is larger than 5 meters and the Kalman filter is not used, the control system could not control the trajectory of the spacecraft. However, a high-precision sensor has a high cost. Therefore, the mission design should consider the cost and benefit of each component taking into account the mission requirements.

5. REFERENCES

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6. RESPONSABILITY NOTICE

The authors are the only responsible for the printed material included in this paper.