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14. Abstract/Notes  <i>The main concern of this work is the establishment of prediction models for the solar activity which greatly influences the behaviour of the upper atmosphere constituent elements. Such models are aimed at applications in orbit computations of close earth satellites (&lt; 1000 km) which, within this altitude range, are affected by atmospheric drag.</i>			
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SOLAR FLUX AND GEOMAGNETIC ACTIVITY PREDICTION

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ABSTRACT

*The main concern of this work is the establishment of prediction models for the solar activity which greatly influences the behaviour of the upper atmosphere constituent elements. Such models are aimed at applications in orbit computations of close earth satellites (< 1000 km) which, within this altitude range, are affected by atmospheric drag.*

1 - INTRODUCTION

The study of solar activity prediction models plays an important role in the area of earth artificial satellites through its influence on orbital trajectories. In artificial satellite orbits, one in general considers accurate dynamic models which, besides the gravitational attraction responsible for the near keplerian motion, include the perturbations due to diverse environmental sources. Particularly in the case of low orbits (< 1000 km of altitude), the atmospheric drag is a very relevant perturbation. The calculation of the atmospheric drag is directly related to the local properties of the atmosphere. Hence atmospheric density models are needed for precision orbit computation. Basically the atmospheric density models are extremely dependent on the temperature [1], [2]. Measurements collected on board of satellites have shown that the local atmospheric temperature at 90 km of altitude is almost constant to 190°K. Nonetheless, from this altitude the temperature increases asymptotically up to the threshold of the exospheric temperature about 600 km. On the other hand, the exospheric temperature is sharply affected by solar radiation, particularly by a narrow band of the spectrum about 10.7 cm of wavelength. The density of the energy flux from solar radiation at 10.7 cm wavelength is usually named F 10.7 solar flux. Daily measurements of the solar flux corrected to one astronomical distance are monthly released by the Herzberg Institute of Astrophysics at Ottawa, Canada, since 1948. The atmospheric density is also influenced, though to a lesser extent, by variations of the terrestrial magnetic field. Among several factors which contribute to those variations, the solar storms are the most significant ones [3]. Such storms increase the number of ionized particles brought about by the solar wind and distort the geomagnetic field lines. The distortion of the field is measured by an index K every three hours by 12 magnetic observatories spread over the world. An average of the local values of K, named planetary index

$K_p$ , is monthly issued by the Institut für Geophysik at Göttingen, Germany. The scale for  $K_p$  is quasi-logarithmic and only discrete values between 0 (minimum activity) and 9 (maximum activity) are utilized. The  $K_p$  values can be converted to

the planetary amplitude  $A_p$  by means of a table whose scale is linear. The index  $K_p$  and amplitude  $A_p$  are both equivalent quantifications of the geomagnetic activity. Because of their effects on the atmosphere, the solar flux and geomagnetic activity prediction models have become necessary for space mission analysis, lifetime calculations and orbit propagations. Though implicitly one acknowledges that those parameters are somehow related to the sunspot number, up to now there is no mathematical model based upon physical relationships linking the causes to the effects. There are only empirical models which combine regression techniques to the polynomial extrapolation [4]. The data bank available for the assessment of the models is considerably dense (daily measurements), but relatively short term (about 37 years) compared to the time constants of the phenomenon. The characteristics of the data can be seen in Figures 1 and 2. The solar flux presents an approximately constant minimum value ( $70 \times 10^{-22} \text{ w/m}^2 \text{ Hz}$ ), perturbed by long term oscillations (~11 years), with amplitudes from 100 to 200. Moreover, irregular short period oscillations whose mean square values (dispersion) increase with the flux intensity are verified; hence, they do present a nearly periodic trend (see Fig. 3). The amplitude  $A_p$  presents similar characteristics though with less pronounced periodicity (see Figs. 2 and 4). Accounting for those basic characteristics, solar flux and geomagnetic amplitude prediction models were developed [5], [6] and are shortly described in the sequel. The complete and explicit formulation is contained in [6], which is to be released forthcomingly.

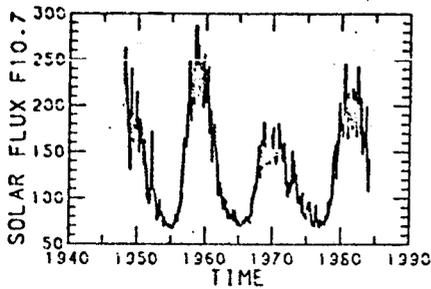


Fig. 1 - Solar flux since 1948.

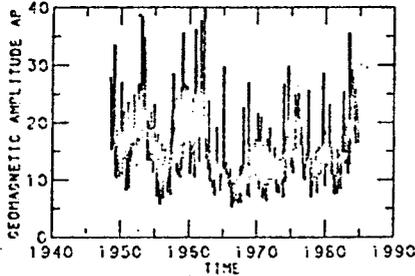


Fig. 2 - Geomagnetic Amplitude since 1948.

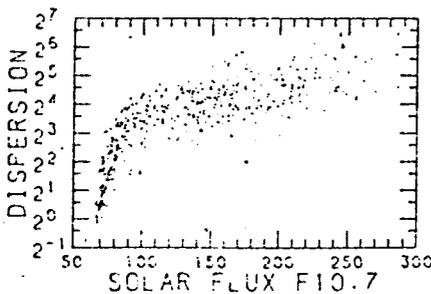


Fig. 3 - Relation between dispersion and solar flux intensity.

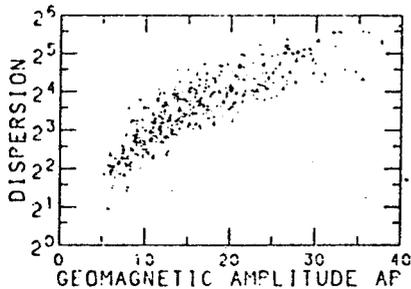


Fig. 4 - Relation between dispersion and geomagnetic amplitude intensity.

## 2 - SOLAR FLUX MODEL

For the solar flux one uses a linear stochastic model, excited by a periodic forcing term, with periodic dispersion as given by:

$$\dot{x} = -\lambda x + u, \quad (1)$$

$$u = U \sin(2\pi t/T + \alpha) + Q^{1/2} \sin(\pi t/T + \beta) w(t), \quad (2)$$

$$F_{10.7} = x + \bar{F}_{10.7} + v, \quad (3)$$

where  $\bar{F}_{10.7}$  is the mean value of the solar flux,  $T$  is the long period,  $w$  is a white noise with unity spectral power density,  $v$  is a white Gaussian process representing the measurement uncertainty, and the parameters  $U$ ,  $\lambda$ ,  $Q$ ,  $\alpha$ ,  $\beta$  are adjusted such that within the steady state the amplitude and the phase of  $x$ , as well as its dispersion, are coincident with the corresponding values of the data bank [6]. The available flux measurements, taken from the data bank, are processed by a linear Kalman filter [7] and time updated according to the dynamic model, Equations 1, 2. The predicted flux as well as its dispersion approach asymptotically the mean periodic values observed in the data bank. However, during a transient about one year, the uncertainty on the propagation remains expressively inferior to the one yielded by the mean periodic behaviour, which is the main advantage of the procedure. Figure 5 shows a prediction from 1964 and the comparison with the actual observed values.

## 3 - GEOMAGNETIC ACTIVITY MODEL

As for the geomagnetic activity amplitude  $A_p$ , one preferred not to use the same type of formulation due to its remarkable lack of uniformity within its cycles. Instead, an approach based upon autoregression linear prediction models was endeavoured [8]. Nevertheless, those models require the knowledge of the correlation between the past measured values with future values to be predicted, but it would only be feasible if  $A_p$  were stationary. Figure 4 suggests the existence of a functional relationship between the dispersion  $\sigma(t)$  and the mean intensity of  $A_p$  as given by:

$$\sigma^2(t) = H[\bar{A}_p(t)], \quad (4)$$

where the function  $H$  might be attained by means of a least square method, and  $\bar{A}_p$  would be the mean  $A_p$ . Although this shows that  $A_p$  is nonstationary, on the other hand it permits to make a nonlinear transformation:

$$A_p^*(t) = G[A_p(t)], \quad (5)$$

such that  $A_p^*$  has almost constant dispersion or, in other words,  $A_p^*$  is stationary. The transformation defined by

$$\frac{dG}{dA_p} = H^{-1/2}(\bar{A}_p), \quad (6)$$

can be deduced by developing  $A_p^*$  in Taylor series up to first order and introducing the hypothesis that the correlation  $R(t, \tau)$  is

$$-R(t, \tau) = \rho(t - \tau) H^{1/2} [A_p(t)] H^{1/2} [A_p(\tau)],$$

$$\rho(0) = 1. \quad (7)$$

Then,  $A_p^*$  results approximately stationary [6] because its correlation is approximately given by  $\rho(t - \tau)$ . Assuming ergodicity of  $A_p^*$ , one can compute  $\rho(t - \tau)$  from the transformed data. The prediction of  $A_p^*$  was accomplished in two phases. In the first, one obtains the mean periodic value for all cycles by means of a periodic smoothing [9], and a spline smoothing [10], which tracks the mean period variations peculiar to each cycle; then the difference between both is propagated (Figure 6 and 7).

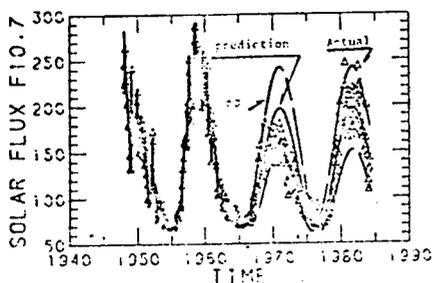


Fig. 5 - Solar Flux Prediction since 1964.

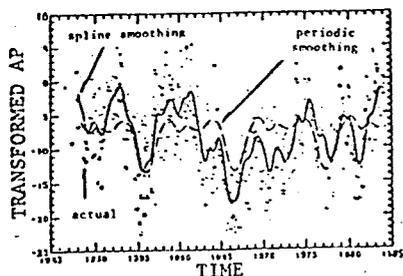


Fig. 6 - Transformed geomagnetic amplitude.

In the second phase, the short period variation regarding the spline smoothing is propagated. Figure 8 depicts a prediction since 1968, obtained by applying the inverse transformation to the predicted values of  $A_p^*$ .

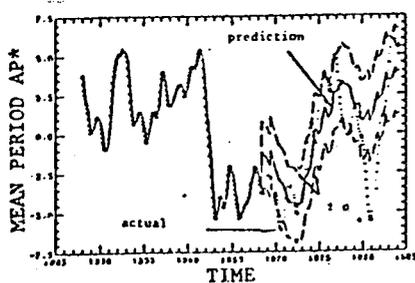


Fig. 7 -  $A_p^*$  mean period prediction since 1968.

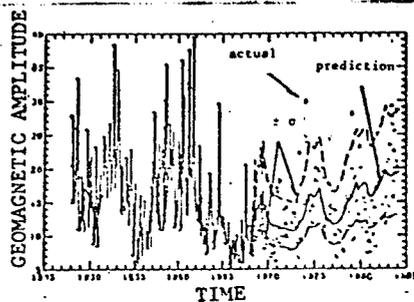


Fig. 8 -  $A_p$  prediction since 1968.

#### 4 - CONCLUSIONS

The comparisons of the results with the actual measured values indicate, for both models, a statistically consistent propagation specially advantageous for predictions around one year. During this transient, the standard deviation is small enough such that reasonable degree of confidence is assured. The model of the sunspot number and its relationship with the solar flux and the geomagnetic activity is a matter of current research. As the available data of solar spot number span an amount of cycles considerably larger, the solar spot model could then be used, as in [4] to enhance long period predictions and, in addition, it would allow meaningful comparisons with their results.

#### 5 - REFERENCES

- [1] JACCHIA, L.G. *Revised Static Models of the Thermosphere and Eosphere with Empirical Temperature Profiles*. Cambridge, MA, SAO, May 1971 (SAO Special Report n° 313).
- [2] \_\_\_\_\_ *Thermospheric Temperature, Density and Composition: New Models*. Cambridge, MA, SAO, March 1977 (SAO Special Report n° 375).
- [3] WERTZ, J.R. *Spacecraft Attitude Determination and Control*. Dordrecht, Holland, D. Reidel, 1978.
- [4] HOLLAND, R.L.; VAUGHAN, W.W. Lagrangian Least-Squares Prediction of Solar Flux ( $F_{10.7}$ ). *J. Geophysical Research*, 89(A1): 11-16, Jan. 1984.
- [5] LOPES, R.V.F.; KUGA, H.K.; CARRARA, V. *Um Modelo de Predição para o Fluxo Solar e para a Atividade Geomagnética*. In: 8th CNMAC, held at Florianópolis, SC, Brasil, Universidade Federal de Santa Catarina, 16-20 Sept. 1985, pp. 123-128.
- [6] LOPES, R.V.F.; KUGA, H.K.; CARRARA, V. *Models for Solar Flux and Geomagnetic Activity Prediction*. São José dos Campos, SP, Brazil, INPE, Dec. 1985. To be published.

- [7] JAZWINSKI, A.H. *Stochastic Process and Filtering Theory*. N. York, Academic, 1970.
- [8] MAKHOUL, J. Linear Prediction: a tutorial review. In: Childers, D., *Modern Spectrum Analysis*. N. York, John Wiley, 1978, pp. 99-118.
- [9] LOPES, R.V.F. *Suavização no Domínio da Frequência*. In 5th CBA, held at Campina Grande, PB, Brasil, Universidade Federal da Paraíba, 3-6 Sept. 1984, pp. 490-495.
- [10] WAHRA, G. Smoothing Noisy Data with Spline Functions. *Numerische Mathematik*, 24: 383-393, 1975.